1. Suppose that missiles have been fired towards a city, and each of them has probability of evading each defensive layer of THAAD batteries: there might be one, two, or more layers. All shelters in the city can effectively withstand at most two hits.  
   1. Suppose there is only one defensive layer of THAAD batteries, and let be the number of incoming missiles that have evaded the defense. Assuming that , derive the probability mass function of as well as the average number of missiles that would evade the defense. (Hint: binomial distribution.)  
        
      .
   2. Suppose there is only one defensive layer of THAAD batteries, and let denote the number of incoming missiles that have evaded the defense. How small should the parameter p be in order for the probability of more than two hits to be smaller than ? (Hint: binomial distribution.)
   3. Suppose there are two defensive layers of THAAD batteries, and let X denote the number of incoming missiles that have evaded the two layers of defense. Assuming that p = 0.05, derive the probability mass function of X as well as the average number of missiles that would evade the defense. (Hint: equation (C1) on p. 75 of our text and two applications of the binomial distribution.)  
        
      Let the probability of evading the first layer be and let the probability of evading the second layer be
   4. Suppose there are two defensive layers of THAAD batteries, and let X denote the number of incoming missiles that have evaded the two layers of defense. How small should the parameter p be in order for the probability of more than two hits to be smaller than 0.001? (Hint: equation (C1) on p. 75 of our text and two applications of the binomial distribution.)
2. The CIA World Factbook says that there are currently (approx.) 35 million Canadians and 324 million Americans. Furthermore, the leading market research company Ipsos MORI says that approximately 86% of Canadians and 83% of Americans are, generally speaking, happy. Suppose that you meet a happy North American, who could be either Canadian or American. What is the probability that the happy person is Canadian? (Hint: you will need the Bayes Theorem on p. 75 of our text.)  
     
   Let be the probability of being Canadian.   
   Let be the probability of being American.   
   Let be the probability of being happy.
3. A certain piece of equipment is regularly checked for failure at times , which could, for example, mean ‘days.’ Let be the failure time, and let denote the corresponding discrete hazard rate function.  
   1. How is defined in terms of the probability ?
   2. Express in terms of the reliability function of ?
   3. It is often more intuitive to first specify a hazard rate function and only then try to see how the corresponding reliability function looks like. For this reason, given a hazard rate function , derive a formula for the reliability function in terms of .  
        
      )  
      )   
      )  
        
      Therefore,
   4. It is often more intuitive to first specify a hazard rate function and only then try to see how the corresponding probability mass function looks like. For this reason, given a hazard rate function , derive a formula for the probability mass function in terms of .
   5. Find of the failure time that follows the geometric distribution, whose probability mass function is for all , where is a constant. Draw the hazard rate function when .
4. Let be the ‘failure time’ random variable that follows the continuous distribution with a probability density function . Let denote the hazard rate function of .  
   1. Express in terms of the reliability function only.
   2. It is often more intuitive to first specify a hazard rate function and only then try to see how the corresponding reliability function looks like. For this reason, given a hazard rate function , derive a formula for the reliability function . What is the reliability function of a hazard whose hazard rate function is constant, say , that is, for all .
   3. It is often more intuitive to first specify a hazard rate function and only then try to see how the corresponding probability mass function looks like. For this reason, given a hazard rate function , derive a formula for the probability density function .
   4. Find of the failure time that follows the exponential distribution with rate , that is, the distribution function of is for all . Draw the hazard rate function when .
   5. Define the lack-of-memory property for any random variable and prove that the property is satisfied by the exponential distribution with rate .  
        
      The lack-of-memory property is a property that eliminates the significance of elapsed time. An RV that has the property does not get effected by elapsed time.
   6. Find the hazard rate function of the failure time that follows the Weibull distribution with parameters and , that is, the distribution function of is for all . Draw the hazard rate function when and .
5. 1. Let denote the ‘filter’ (that is, the random variable) that takes the value 1 when event occurs and 0 otherwise. Prove that if two events and are independent, then the expected value of the product is the product of the expected values of and . (Hint: page 96.)  
        
       and are independent, then,
   2. Let and be two discrete random variables. Prove that if these random variables are independent, then the expected value of the product is the product of the expected values of and . (Hint: page 101.)
   3. Prove that if two random variables and are independent, then the random variables are uncorrelated, that is, their covariance is zero. (Hint: page 102.)  
        
       and are independent, then
   4. Prove that if two random variables and are independent, then the variance is the sum of the variances and . (Hint: page 102.)